

Some Investigations on the Time-of-Flight (TOF) array Multiple Coulomb Scattering (MCS) on tracking and momentum resolution.

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Abstract – The current MIPP geometry, chamber resolutions and magnetic field values in the “Rosie spectrometer” limit the momentum resolution to $\delta p_z/p_z = 0.0034 p_z$, largely due to the error in determining the slope of the trajectory downstream of Rosie. The addition of the Time-of-Flight array will increase the slope determination by scattering the tracks after the upstream slope is measured. The change of slope due to the multiple scattering is of order the ideal slope uncertainty at 100 GeV/c, and is two orders of magnitude larger than the slope uncertainty at 10 GeV/c. The fractional error in the momentum determination is larger at high momentum.

Introduction and Outline

The MIPP layout can be considered as two spectrometers for the purpose of track reconstruction. The Jolly Green Giant (JGG) spectrometer consists of the JGG analyzing magnet, the TPC and the drift chambers 1,2 and 3. The Rosie spectrometer consists of drift chambers 1, 2, 3 and 4 (the “E690 chambers”), and the MWP Chambers 5 and 6 (the “Iowa chambers”). Based on past experience, the JGG spectrometer is capable of high momentum resolution up to a maximum particle momentum in the neighborhood of 15 to 20 GeV/c. The Rosie spectrometer is present to measure the high momentum particles, where “high” starts around 10 GeV/c and goes to 120 GeV/c. The RICH windows limit the acceptance of the Rosie spectrometer.

Evaluating the effect of MCS from the TOF array on the resolution of tracks in the MIPP spectrometer can be performed using an idealization of track reconstruction for the Rosie spectrometer system. This idealization makes use of the approximation that the z-component of the momentum of the tracks is inversely proportional to the change in the x-z slope of the trajectory at the midpoint of Rosie due to its “ p_T kick”. The reconstruction problem reduces to fitting the straight line segments upstream and downstream of the Rosie magnets. Upstream segments are made from points in chambers 1, 2 and 3; downstream segments in chambers 4, 5 and 6.

For this study it is possible to assume the current MIPP geometry, the historically achieved chamber resolutions and a track stepper (see the note “Charged Particle Tracking Through Magnetic Fields”). For the moment I have considered only unvarying B_y fields in the magnet. The chambers are also idealized as space points. In chambers 1, 2, 3, and 4 the four signal planes provide information regarding the x-z trajectory of the track. Finally, I have not included any other MCS sources. The air paths can have significant effect on trajectories. In E690 for instance, all spectrometer paths were either in material, chamber gas or He. It is not clear that the physics goals of MIPP will be compromised by this choice.

Fitting

This memo side steps the issues around track finding and assumes that for a given trajectory the (x,y) coordinate is known at a particular chamber. The track parameterization is a simple line:

$$x(z; x_R, x') = x_R + x'(z - z_R)$$

where the coordinate system is taken in the center of the Rosie magnet, thus the “R” subscript. Fitting proceeds by minimizing the χ^2 with respect to the parameters (x_R , x'):

$$\chi^2 = \frac{1}{N_{\text{dof}} - 1} \sum_i \frac{1}{\sigma_i^2} [x_i - x_R - x'(z_i - z_R)]^2$$

the weighted sum of the square of the difference between the i^{th} measured point and “predicted” position. The value of σ_i is the error associated with the i^{th} measurement. The minimization is straight forward and follows from solving two equations with two unknowns:

$$\Delta z_i = z_i - z_R$$

$$\frac{\partial \chi^2}{\partial x_R} = 0 = \sum_i \frac{1}{\sigma_i^2} [x_i - x_R - x' \Delta z_i]$$

$$\frac{\partial \chi^2}{\partial x'} = 0 = \sum_i \frac{1}{\sigma_i^2} \Delta z_i [x_i - x_R - x' \Delta z_i]$$

for which the classical result obtains:

$$x_R = \frac{1}{\sum_i \frac{1}{\sigma_i^2} \sum_i \frac{\Delta z_i^2}{\sigma_i^2} - \left(\sum_i \frac{\Delta z_i}{\sigma_i^2} \right)^2} \left[\sum_i \frac{\Delta z_i^2}{\sigma_i^2} \sum_i \frac{x_i}{\sigma_i^2} - \sum_i \frac{\Delta z_i}{\sigma_i^2} \sum_i \frac{x_i \Delta z_i}{\sigma_i^2} \right]$$

$$x' = \frac{1}{\sum_i \frac{1}{\sigma_i^2} \sum_i \frac{\Delta z_i^2}{\sigma_i^2} - \left(\sum_i \frac{\Delta z_i}{\sigma_i^2} \right)^2} \left[\sum_i \frac{1}{\sigma_i^2} \sum_i \frac{x_i \Delta z_i}{\sigma_i^2} - \sum_i \frac{\Delta z_i}{\sigma_i^2} \sum_i \frac{x_i}{\sigma_i^2} \right]$$

the variance on the parameters fit in this manner is taken to be inverse of the square root of the appropriate element of the diagonal of the covariance matrix:

$$\sigma_{x_R} \approx 1 / \sqrt{\sum_i 1 / \sigma_i^2}$$

$$\sigma_{x'} \approx 1 / \sqrt{\sum_i \Delta z_i^2 / \sigma_i^2}$$

For a given trajectory, the parameters can be determined and an error assigned to the determination. This is done for chambers upstream and downstream of Rosie. For a given trajectory the two segments should intersect at some z inside of the Rosie field (the effective midpoint).

To first order, the trajectory simply changes slope as it moves through the field. This is a “thin lens” approximation to the magnetic optics. The downstream slope is given by the expression:

$$x'_{\text{ds}} \approx x'_{\text{us}} + q \frac{p_{\perp R}}{p_z}$$

where q is the charge of the particle, p_z the z -component of the particle’s momentum and $p_{\perp R}$ is the Rosie “momentum kick”. The slopes determined by fitting allows the momentum to be calculated given the Rosie field (expressed in momentum):

$$p_z = \frac{p_{\perp R}}{x'_{us} - x'_{ds}}$$

$$\delta p_z = p_z \sqrt{(\delta x'_{us}/x'_{us})^2 + (\delta x'_{ds}/x'_{ds})^2}$$

the second equation giving the estimated error of the momentum determination.

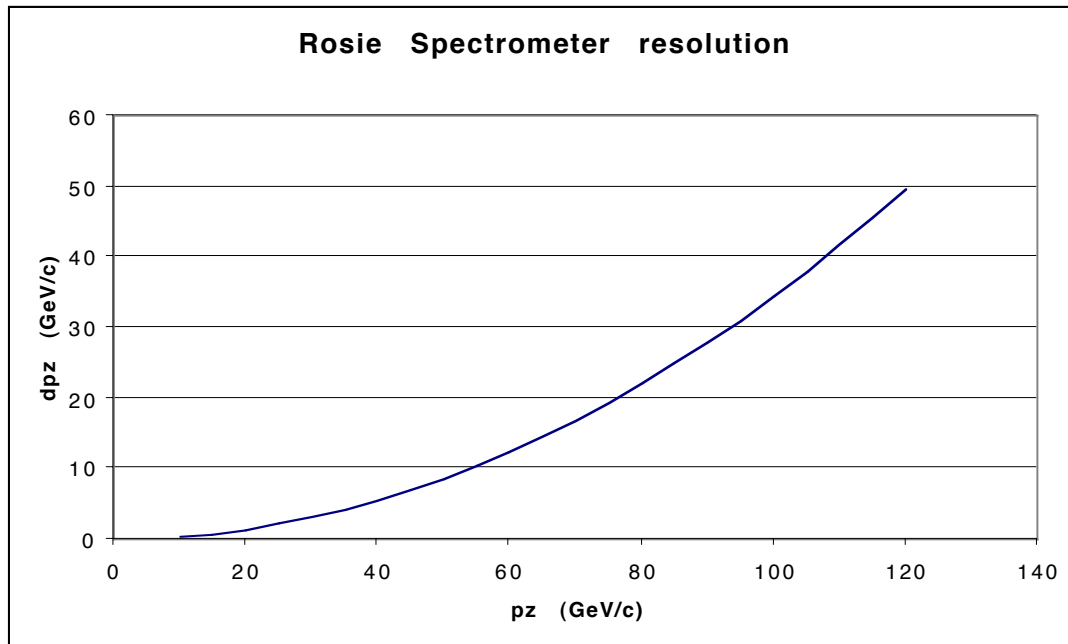
An Excel spread sheet was used to track the trajectory through the MIPP geometry. The x-positions of the trajectories were used as input to determine the track parameters. The historical resolutions of the chambers (250 μm for the E690 chambers and 870 μm for the Iowa chambers) were used in the fit and subsequent error calculation.

Analysis

The Multiple Coulomb Scattering was put into one of the worksheets in the spreadsheet at the TOF location. The thickness of the scintillator slab could be varied and the effect on the momentum determination and the spatial position of the trajectory observed.

The first result is that the downstream momentum resolution is limited by the fractional error in the determination of the slopes. This gets worse with decreasing slopes as expected. The plot below is described by the expression:

$$\frac{\delta p_z}{p_z} = 0.0034 p_z [\text{GeV}/c].$$



The error in the slope is of order a few 10^{-5} both upstream and downstream. MCS will “add” to the slope at the TOF position. For a 10 cm thick slab of scintillator the multiple scattering angle is 5×10^{-5} for a 115 GeV/c particle. This is of the same order of magnitude as the slope error. At 10 GeV/c the MCS scattering angle is an order of magnitude larger and dominates the error in the momentum reconstruction (the slopes are of order 10^{-2} at these momentum thus the fractional error is quite small).

At the lower momentum the trajectories can be displaced in chamber 6 by as much as a centimeter from the non-scattered position. Large displacements due to MCS will require a large search area during the track finding phase of the reconstruction.

The high momentum resolution can be improved by placing a higher resolution chamber near chamber 6. For a $50\mu\text{m}$ spatial resolution chamber the momentum resolution would be $2.27\text{ GeV}/c$, or 2.3% . This would also require a hole in the TOF array through which these high momentum particles could pass without scattering. At $50\text{ GeV}/c$ a hole $\pm 15\text{ cm}$ in x and $\pm 11\text{ cm}$ in y would allow particles with a $p_{\perp} \leq 1\text{ GeV}/c$ through.

High resolution chambers, such as the E690 beam chambers, exist but have limited aperture sizes. A more detailed study of the desirability of the high momentum resolution requirements of MIPP must be undertaken to answer some of the questions raised by this first, quick look.